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ON SOME PHYSICAL PROPERTIES OF ICE

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INTRODUCTORY NOTE BY T. C. CHAMBERLIN

While many of the more obvious problems of ice and ice action have been solved in a general way, there remain not a few questions of a more refined sort which require solution before glaciology can rest on a secure foundation. Some of these questions are critically important for they bear radically on interpretations that have already been widely accepted and are currently taught. More extended and more critical field studies are required to solve some of these questions while the solution of others depends on more discriminative and exact laboratory experimentation. All of them call for more searching analyses of the problems themselves, as a source of guidance in field work and in experimentation, as also in the interpretation of results. The glacialists working at Chicago have been trying to do their bit toward the solution of some of these problems and have had under way for some time a series of attacks along several lines in both field and laboratory. This paper presents the preliminary results of a careful series of laboratory determinations carried out by Professor Motonori Matsuyama, of the Department of Geophysics of the Kyoto Imperial University, Japan, who has been spending the year at Chicago.

When McConnell, followed by Mügge, announced that ice crystals are minutely laminated in planes normal to their optical axes and that movement along these planes was notably easier than in other directions, it was felt by many that these disclosures offered a happy solution of the anomaly of glacial movement which seemed to be a quasi-fluidal flow in a body obviously rigid. But later critical studies raised serious doubts as to the actual participation of the gliding planes in ordinary glacial motion, and so the subject came to demand more refined examination. Up to date, no one, so far as I know, has determined what is the measure of the resistance to motion along these planes compared with the stresses actually brought to bear upon them in ordinary glacial motion. Nor has it been shown whether the relation of these gliding planes to one another is of the elastic or the viscous order. But even if these properties were known, there would still remain the radical question whether glacial motion actually takes place by means of movements along these planes—or in any other way *within* the constituent crystals—or whether it is essentially a motion between the constituent crystals. There are here therefore two quite distinct problems. The investigations of Professor Matsuyama relate to the second of these.

The method of Matsuyama is, so far as I know, unique in that he deals with bundles of crystals which have a common known orientation. He thus brings movement along the gliding planes into experimental competition with movement between the crystals. All are familiar with experiments upon the yield of glacier ice where the mass under test was formed of many crystals of *diverse* orientation, but this very diversity of orientation stood in the way of a strict interpretation of the results. Matsuyama, however, so selected his prisms and cylinders as to force an alternative between combined movement on gliding planes and movement between the crystals. His method and his achievement are therefore notable.

It is further to be observed that in his experimentation he used the torsional method, following Michelson, in which the errors arising from the stretching or compression of the prisms or cylinders are avoided, since the cross-section remained constant throughout the trial. But as check upon the results of torsion, Matsuyama added the method of bending in which stretching on one face and compression on the other were involved. In the interpretation of the results of this method he called in the resources of the petrographic microscope with the discriminating results given in the text.

INTRODUCTION

The motion of an ice sheet along the mountain slopes and over a large area of the Continent may be caused by more or less different forces. Besides the external forces, it is also important to know what is the behavior of the ice itself in such motion. Numerous works have been published on these problems, among which those of McConnell and Mügge are famous and have been referred to by many authors. According to them an ice crystal can be sheared more easily in the direction parallel to the basal plane than in any other direction. The elaborate works of the members of the Cornell University geological staff[1] added important contributions to the same line. Their idea is that the gliding planes of ice crystals arranged parallel to their basal sections control the behavior of an ice mass as the main factor.

Deeley[2] calculated the viscosity of ice from Main's experiment and found its value to be 6×10^{12} c.g.s. at 0° C., while his own observations on Swiss glaciers gave 125×10^{12} c.g.s.

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[1] R. S. Tarr and J. L. Rich, *Zeits. f. Glets.*, Vol. VI (1912), pp. 225–49; R. S. Tarr and O. D. von Engeln, *Zeits. f. Glets.*, Vol. IX (1915), pp. 81–139; O. D. von Engeln, *Amer. Jour. Sci.*, Ser. 4, Vol. XL (1915), pp. 449–73.

[2] R. M. Deeley, *Geol. Mag.*, New Ser. 5, Vol. IX (1912), pp. 265–69.

Various investigations related to glacier problems were planned and worked on for many years in this department by Professors T. C. Chamberlin and R. T. Chamberlin. The present work is a part of that series and gives the result of preliminary investigations on the elastic properties of ice. Since torsional force applied to a circular rod is the only type of strain, as Professor Michelson[1] says, in which the cross-section remains constant, the main part of the present study consists of observations on torsional deformation. Later some attempts were made to determine the Young's modulus and also to observe what would happen in bending.

The preliminary work was begun early in the autumn of 1919, but owing to a delay in securing certain necessary apparatus, it was not until about the beginning of February, 1920, that the work really began in earnest. About the end of March it became so warm that it was no longer possible to continue the work on ice. Because the time available for this cold work was so short, the results are to be considered only as preliminary; yet they were so suggestive that the present writer considered it worth while to publish them even with these limitations.

During the research, Professor R. T. Chamberlin took constant interest and gave important suggestions for which the writer is very much obliged.

LIMIT OF ELASTICITY

According to Professor Michelson,[2] the deformation of an elastic body passes through four different stages as it goes on, and in the first portion the deformation is characterized by being approximately proportional to the stress. The limit of this part is well defined in some materials while in others it is not so distinctly marked.[3] The following description will show that ice also has a rather well-defined limit of elasticity in some cases at least.

Method.—In the present experimentation a circular rod of ice was held horizontally with its one end fixed to a rigid stand and the other end attached to a circular disk of radius 7.45 cm. movable around a knife-edge at its center which rested on a horizontal plane. The whole equipment was kept in a thermos box, the temperature inside of which could be kept low and constant by using freezing mixture. The torsional stress was applied by putting weights on the scale pan hanging from the limb of the circular disk through the bottom of the box. The amount of torsion was observed by means of a telescope through a window in one side of the box.

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[1] A. A. Michelson, *Jour. Geol.*, Vol. XXV (1917), p. 405.

[2] *Ibid.*

[3] A. E. H. Love, *The Mathematical Theory of Elasticity*, p. 112.

In the first set of experiments, the test piece was cut out from artificial ice and consisted of parallel crystals with their optic axes transverse to the longer dimension of the bar. The first bar was 13.15 cm. long and 1.73 cm. in mean diameter.

The deformation was found to depend upon the rate of increase of force, especially near and beyond the elastic limit. In the first experiment the force was increased by adding two pieces of weights, each weighing 4.40 gm., every five minutes. The total time duration for this observation was 3.5 hours and meanwhile the temperature was constant at $-7^\circ\!.0$ C. The observed relation between the weight and deformation is shown in Figure 1.

From this curve one can easily see that a rather marked change of yielding is recognizable at the point *B*, up to which the strain is more or less proportional to the stress. This feature is none but the characteristic of an elastic curve[1] and we may say that ice behaves like an elastic solid.

Later it was learned that the point *B* was not so well defined in some cases as in the present one. It is for further study to see when this point will be sharply shown and when not.

MODULUS OF RIGIDITY

Rigidity is the resistance of a material to shearing force and is given by the ratio of the amount of shear to the applied force. If a circular rod of length l and radius a is twisted through an angle Φ by an external couple PL, the modulus of rigidity n will be given[2] by

$$PL=\tfrac{1}{2}\pi n\frac{\Phi}{l}a^4.$$

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[1] Given in every book on elasticity.

[2] Poynting and Thomson, *Properties of Matter*, p. 79.

It was found that it took about 10 minutes to reach the new equilibrium position when an ice bar is twisted by a certain stress within the elastic limit. The deformation in the foregoing observation in the part *AB*, therefore, must be slightly modified for this effect. Careful observations were made, increasing the load by the same amount at intervals of 20 minutes. As the result it was found that the mean deviation was $5'.2$ per 26 gm. on the scale pan, the temperature being kept at $-7^{\circ}.5$ C. From this and other known values of the constants, we obtain

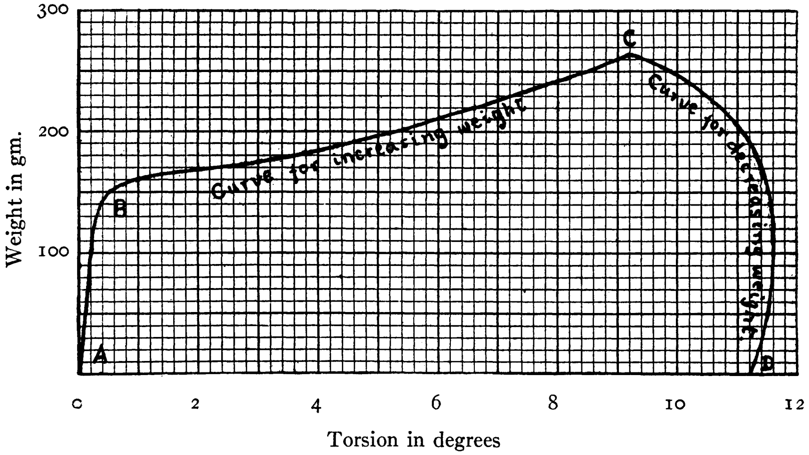

FIG. 1.—Torsion curve of an ice bar, 13.15 cm. long and 1.73 cm. in diameter, with crystals transverse to its longer dimension. Arm of the torsional couple 7.45 cm. Temperature $-7^{\circ}.0$C.

$$n = 1.9 \times 10^{6} \text{ c.g.s.}$$

The value of n was calculated from data in several experiments intended primarily for other purposes. The following table gives the values of n thus obtained for ice bars with optic axes of their component crystals transverse to them.

Temp. (C.)	$n \times 10^{-6}$	Temp. (C.)	$n \times 10^{-6}$
−11.7	2.0	−9.3	2.1
−11.7	2.0	−7.5	1.9
−10.0	1.6	−6.7	1.6
−10.0	2.1	−6.7	1.6
− 9.5	2.0	−4.0	1.6

These results seem to show some decrease of rigidity with rising temperature. Near the melting-point of ice, its rigidity will decrease more rapidly than at lower temperatures, so that the rigidity-temperature curve will be concave toward the temperature axis. If we consider rigidity to depend upon the temperature t up to the second power and calculate the coefficients from the foregoing data, we obtain

$$n=(0.18-0.095\,t-0.0020\,t^2)\times 10^6 \quad \text{c.g.s.}$$

This is represented by the curve in the following figure (Fig. 2), the observed points being denoted by crosses.

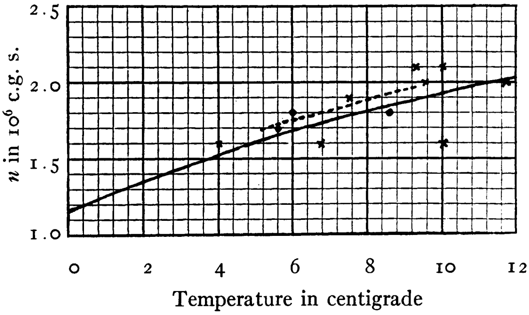

FIG. 2.—Modulus of rigidity of ice composed of parallel crystals at various temperatures. Crosses denote the values when shear is parallel to the optic axes of the crystals and dots the values when shear is parallel to their basal sections.

Similar observations were made for ice bars with the optic axes of the constituent crystals parallel to the axis of the bar. The test piece was 20.95 cm. long and 1.93 cm. in diameter. The crystals were for the most part several millimeters in diameter and never so long as to reach from one end of the bar to the other. The mean deviation was 10′ by 44.0 gm. at −6°.0 C. Using the known values of the constants, the rigidity was calculated to be

$$n=1.8\times 10^6 \text{ c.g.s.}$$

The following table gives the value of n determined from different experiments:

Temp. (C.)	$n\times 10^{-6}$
−9.2	2.1
−8.6	1.8
−6.0	1.8
−5.6	1.7

These data are, by no means, sufficient to determine the relation between the temperature and the rigidity, or to compare this case with the former. As a very rough approximation, however, if we take the means of the first two and the last two and join the points corresponding to them in the figure, we will see that the joining line lies above the curve for the ice bar with transverse crystals. This at least suggests that an ice mass will be deformed more easily when it is twisted parallel to the optic axes of the constituent crystals than when twisted parallel to their basal sections. This result alone is not much to be relied upon. But it is consistent with the results which will be described later.

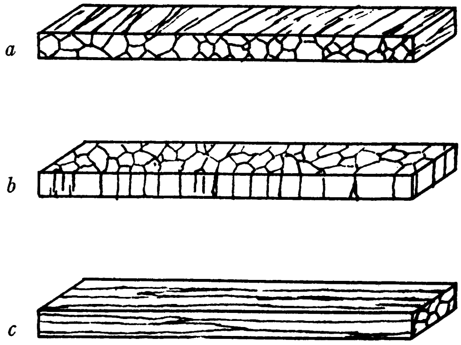

Fig. 3.—Orientations of crystals in test specimens for bending

YOUNG'S MODULUS

Elastic behavior of a solid is determined by two independent elastic constants. We have already found the value of the modulus of rigidity of ice. It is desirable to find also the other, or Young's modulus, which is the resistance of a wire to elongation defined as the ratio of the stress to the strain. To determine this, the usual method of bending was used, with two mirrors and the scale-and-telescope method.[1]

The test specimens were bars of ice with rectangular cross-section (Fig. 3). The component crystals were arranged either (*a*) horizontally or (*b*) vertically transverse to the bar, or else

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[1] Poynting and Thomson, *Properties of Matter*, p. 100.

(*c*) parallel to the length of the bar. Since, as we will see later, ice is affected by the phenomena of elastic fatigue, a new bar was generally prepared for each experiment. These bars were not always of the same magnitude; the bar 1*a* was made by cutting bar 1 in half to see if we could get the same result. The results of several observations are given in the following table:

Specimen No.	Orient.	Length cm.	Width cm.	Thickness cm.	Temp. (C.)	Young's Mod. 10^6 c.g.s.
1	*a*	31.0	1.58	.95	−5.0	11.2
1*a*	*a*	15.0	1.58	.95	−4.8	10.5
2	*a*	18.0	1.87	1.05	−2.0	6.0
3	*b*	22.5	1.73	.85	−3.0	6.0
4	*b*	18.0	1.80	1.02	−2.2	5.9
5	*c*	22.2	1.62	.83	−4.0	18.9
6	*c*	11.0	1.25	.76	−4.1	16.6
6*a*	*c*	11.0	1.25	.76	−3.0	20.0

It will not be safe to consider the effect of temperature upon Young's modulus in each case from these few determinations. Since the range of temperature in these observations is not very large, we may consider the mean values for each case not very far from the truth. These mean values are:

Orient.	Temp. (C.)	Young's Mod. 10^6 c.g.s.
a	−3.9	9.2
b	−2.6	6.0
c	−3.7	18.5

Here it is clearly shown that the specimens of orientation *c* whose crystals are parallel to the long dimension of the bar have the largest value of Young's modulus. We have already seen that the modulus of rigidity seemed greater when shear is parallel to the basal section of the constituent crystals than when perpendicular to it. These results have very important meaning in considering the behavior of an aggregate of ice crystals. If the gliding planes of an ice crystal parallel to the basal plane are the main factor determining the behavior of an aggregation of ice crystals under deforming forces, we must expect that ice will be deformed most easily when the deforming force is parallel to the basal planes of the constituent crystals. The present result shows the reverse fact which suggests that there must be some other factor controlling the deformation.

It is interesting to compare the values of elastic constants of ice with those of other materials.[1]

Material	Rigidity	Young's Mod.
Steel	8.12×10^{11}	20.9×10^{11}
Lead	0.56×10^{11}	1.62×10^{11}
Glass	$3.\times10^{11}$	$7.\times10^{11}$
India rubber	1.6×10^{7}	$5.\times10^{9}$
Ice (*a*)	1.6×10^{6}	9.2×10^{6}
(*b*)		6.0×10^{6}
(*c*)	1.8×10^{6}	18.5×10^{6}

Thus we see that ice has much smaller elastic constants than metals and very small even compared to india rubber. For metals the value of Young's modulus is generally about three times as large as that of rigidity. For india rubber it is about three hundred times as great. For ice, the value of Young's modulus is from four to ten times as great as the rigidity, depending upon the orientation of the constituent crystals.

TORSION BY CONSTANT FORCE

As long as it remains within the limit of perfect elasticity, the deformation will little depend upon time. After that it will take considerable time before the final position of equilibrium is reached under a given force. For the purpose of finding the mode of yielding by constant force, a circular rod of ice with crystals transverse to it was twisted by different amount of torsional couple. The rod was 20.85 cm. long and 1.96 cm. in mean diameter. The torsional angle was observed at various times after a certain amount of weight was put on the hanging scale pan. The yielding curves under different weights are given in Figure 4.

As to the law of elastico-viscous flow of solids, Professor Michelson has published the results of his elaborate work with a formula which is to be used for materials of widely different properties.[2]

[1] C. W. C. Kaye and T. H. Laby, *Physical and Chemical Constants*, p. 27.

[2] A. A. Michelson, *Jour. Geol.*, Vol. XXV (1917), pp. 405–10, and Vol. XXVIII (1920), pp. 18–24.

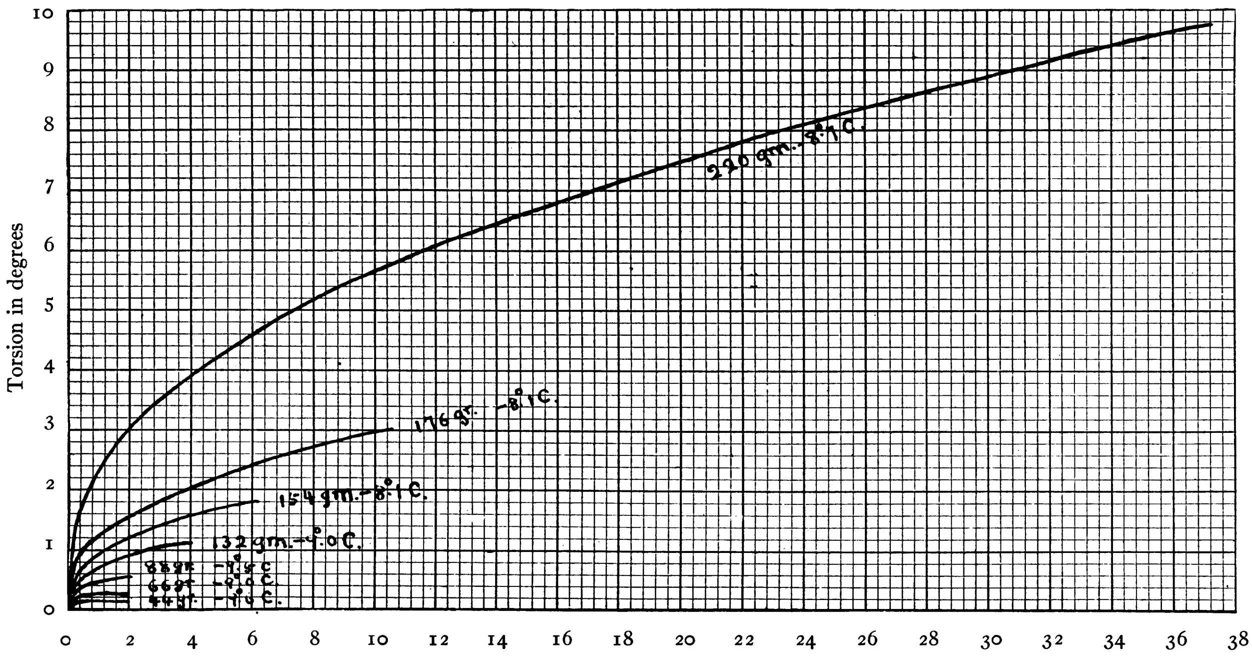

Fig. 4.—Torsion curves, by constant forces, of a circular ice bar, 20.85 cm. long and 1.96 cm. in diameter with crystals transverse to its longer dimension.

If we calculate the values of constants for that formula from the foregoing data, we can obtain the final value of torsion for each weight. Since the writer expects to obtain more detailed data for ice by further experiments, he will be, for the present, satisfied by finding the final positions by graphical extrapolation. The relation between the torsion and the applied force, thus found, is given in Figure 5. This curve is not the same as the curve in Figure 1, where the mode of application of force was different from the present case, as were also the dimensions of the test piece.

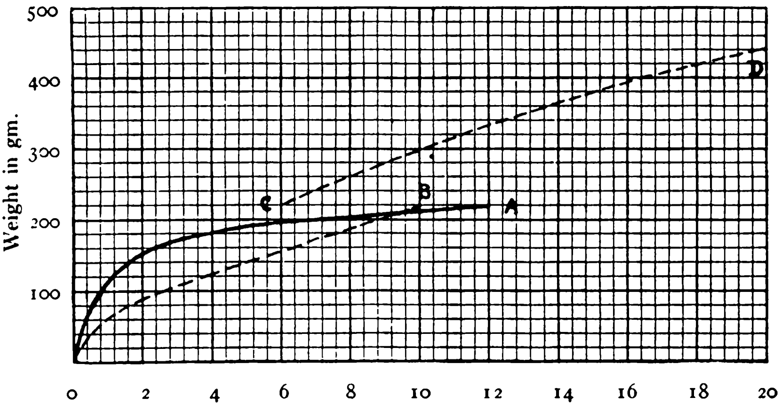

Fig. 5.—Torsion curves obtained from Figures 4 and 6. Full line: for the bar with crystals transverse to its longer dimension. Broken line: for the bar with crystals parallel to its axis.

Similar observations were tried on a test piece with crystals parallel to the length. It was 20.95 cm. long and 1.93 cm. in diameter (Fig. 6). During this test, it was noticed that the twisting curve for 308 gm. was very close to that for 220 gm. When repeated with 220 gm. once more, the deviation curve was found to be much lower than the former curve for the same amount of force. This fact suggests that ice shows the phenomena of elastic fatigue.[1] It was not learned whether this occurred after repetition of small deformations or after deformation of certain amount. It seems likely that this phenomena appears rather abruptly instead of gradually.

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[1] Poynting and Thomson, *Properties of Matter*, p. 57.

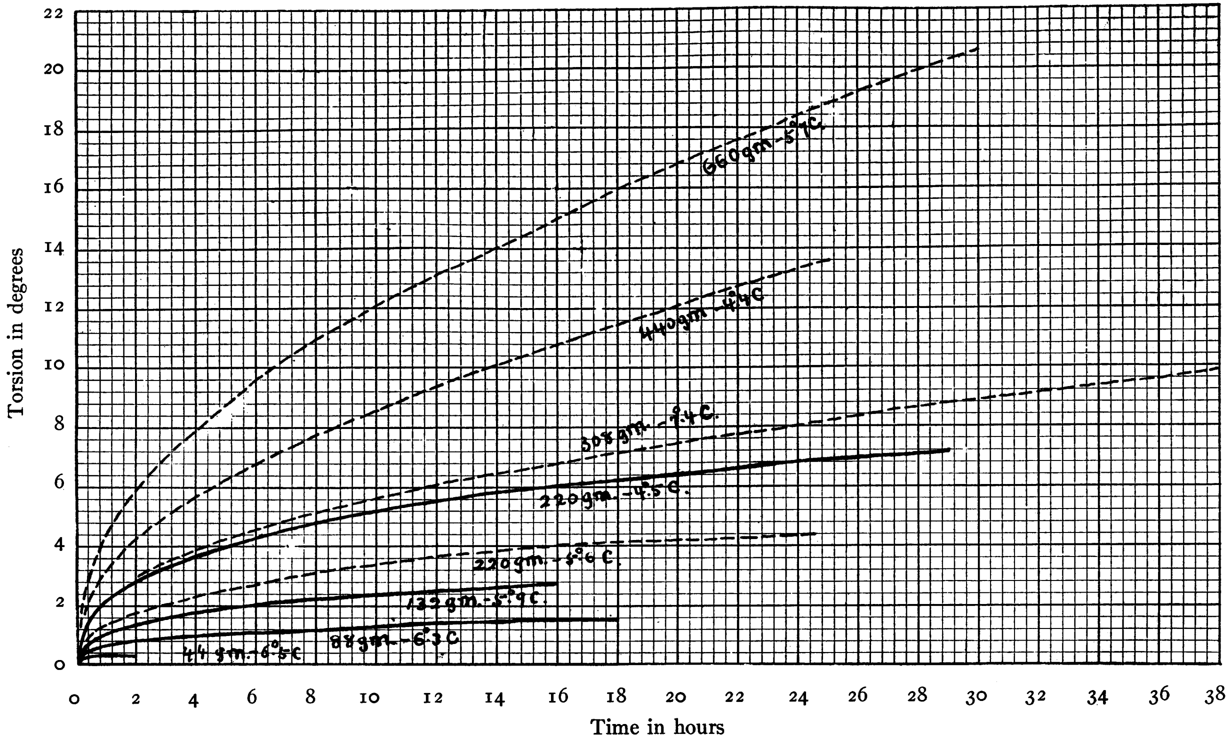

Fig. 6.—Torsion curves, by constant forces, of a circular ice bar, 20.95 cm. long and 1.93 cm. in diameter with crystals parallel to its axis.

The torsion curve for the case with crystals parallel to the axis of the test piece was also obtained as on the other case and is shown in Figure 5 by the broken line. Since the test specimens for these two cases were of nearly the same dimensions, we can compare the two curves with each other and consider the relative ease of yielding to torsional stress. As long as the deforming force remains less than 200 gm. the deformation for the former case is much smaller than for the latter. After that the curve becomes very flat showing that the specimen begins to yield readily. The curve for the latter rod shows a discrepancy between the points *B* and *C*. After that it still keeps on its steepness, becoming flatter after the point *D*, though this is not shown in the figure. Attention must be called to the fact that the observations for the former rod were made after it had been used repeatedly for other tests. It is, therefore, probable that this rod was already affected by the elastic fatigue when the present series of observations began. Lacking the knowledge how far this is the case, it is not safe to compare the curves for weights of less than 200 gm. When the weight is greater than this, it seems quite probable that the former rod, in which the constituent crystals are lying transverse to it, is twisted more easily than the latter, in which they are parallel to the length of the rod.

TORSION BY INCREASING FORCE

Whatever may be the fundamental property of ice by which the glacial motion takes place, the rate of accumulation of snow must be one of the most important factors influencing that motion. The corresponding study in the laboratory should be to find the relation between the deformation and different rates of increase of deforming force. For this purpose, a series of observations has been made with the same two test rods of different orientations of crystals as before.

The observed deformations of the first rod, where the crystals were arranged transverse to the rod, are shown in Figure 7. Since the deformation of a plastic body under constant force increases with time, it is clear that when the force increases slowly, the deformation for the same amount of force will be greater than when the force is increased rapidly. This relation is clearly shown in the figure, the curve being steeper when the force is increased more rapidly. One curve, when the force was increased by the rate of 1.76 gm. per minute, is exceptional and steeper than the curves for the rates of 4.4 gm. and 2.2 gm. per minute as is shown by the broken line. The other curve for the same rate, which is not given in the figure, was very close to and slightly steeper than the curve for the rate of 2.2 gm. per minute. Since these two curves were obtained by the last observations of the present series, the foregoing fact may be considered as the result of elastic fatigue. The difference between these two curves for the same rate of increase of force may be due to the difference of temperature which was respectively −10°.4 C. for the steeper and −4°.6 C. for the flatter.

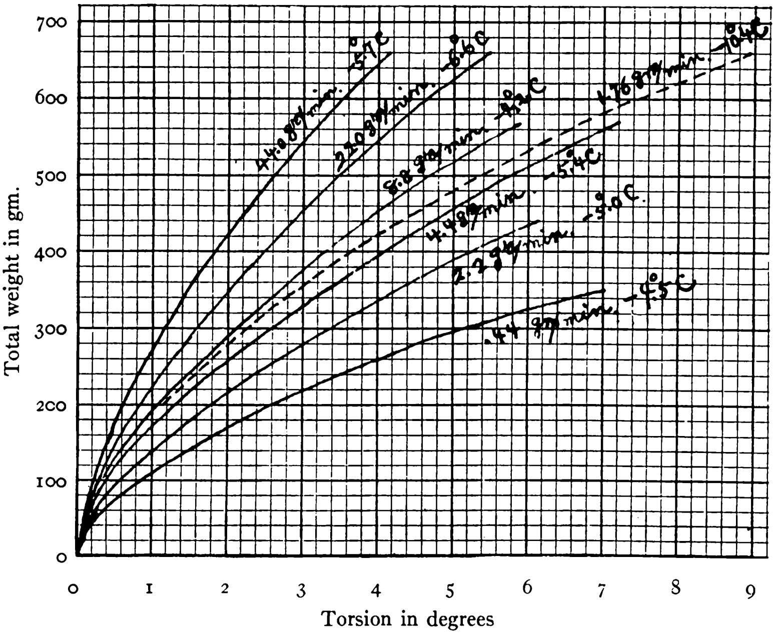

FIG. 7.—Torsion curves, by increasing force, of the first bar used for torsion by constant force.

Similar curves, though less numerous, were obtained with the second rod, in which the crystals were parallel to the length of the rod. These are shown in Figure 8. Here the deformation for the rate of 1.76 gm. per minute was observed before it was used for the series of experiments on torsion by constant force in which the effect of elastic fatigue, as previously stated, was observed in pronounced form. The other two curves were obtained after that, and hence it is possible that the flatness of the first curve compared to the curve for .88 gm. per minute, which should not be the case, may be due to fatigue.

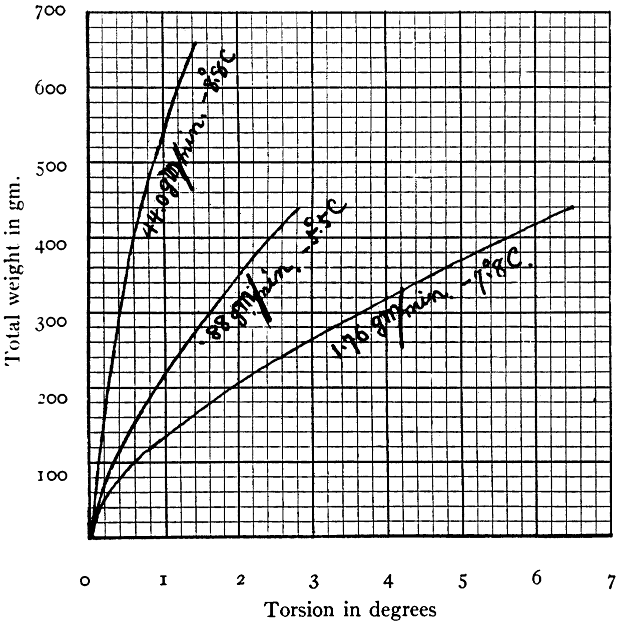

FIG. 8.—Torsion curves, by increasing force, of the second bar used for torsion by constant force.

The curves in Figure 7 were obtained after the observations for the torsion by constant force. We have seen before that the test piece was already subjected to fatigue in that case, and consequently the present curves will be affected in the same way. As a very rough approximation, therefore, we may compare the curves for the rates of .88 gm. and 44.0 gm. per minute in Figure 8, assuming the effect of fatigue to be the same after it has abruptly appeared and also neglecting the effect of temperature, which did not differ very much. The former curves are so much steeper that this approximation seems safe. This steepness of the former curves means that ice is more easily twisted when the constituent crystals are arranged transverse to the rod than when they are parallel to its axis.

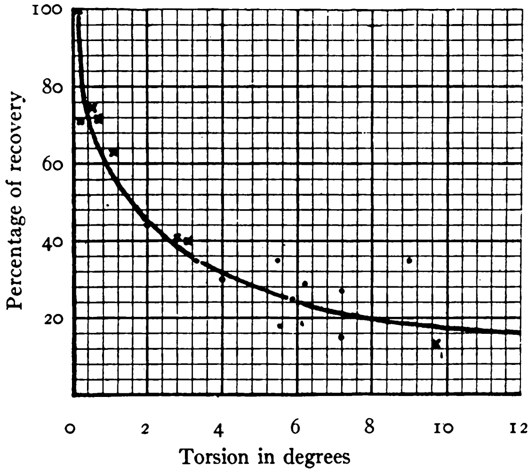

FIG. 9.—Recovery curve for the rod with crystals transverse to its longer dimension. Crosses for constant force and dots for increasing force.

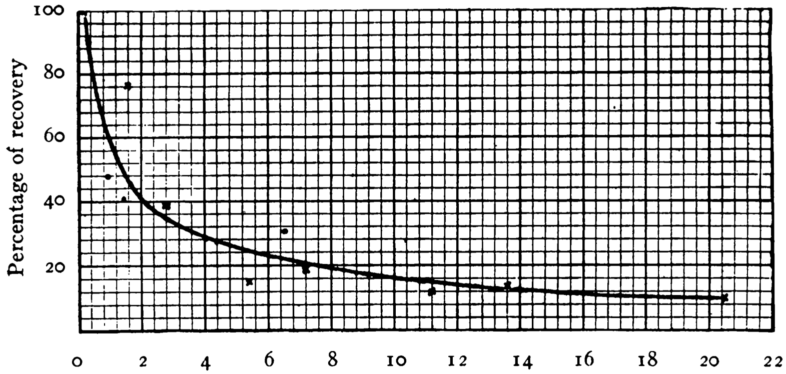

FIG. 10.—Recovery curve for the rod with crystals parallel to its axis. Crosses for constant force and dots for increasing force.

RECOVERY FROM DEFORMATION

While observing the deformations by constant force as well as by increasing force, the amount of return toward the original state after removal of the force was observed in each case. Beside the amount of torsion, the recovery will depend upon various other conditions, like the temperature, mode of applying the force, and its duration. These values are given in the following tables:

CONSTANT FORCE. CRYSTALS TRANSVERSE

Temperature	Weight	Duration	Deformation	Recovery	Percentage
−9°.0C.......	22 gm.	0^h15^m	$0^\circ\ 4'$	$0^\circ\ 4'$	100
−9.6........	44	1 0	0 8	0 8	100
−9.0........	66	1 0	0 14	0 14	71
−9.5........	88	2 0	0 32	0 24	75
−9.0........	110	3 5	0 36	0 26	72
−9.0........	132	3 45	1 5	0 41	63
−8.8........	154	6 20	2 50	1 2	41
−8.1........	176	10 39	3 2	1 12	40
−8.7........	229	37 20	9 44	1 18	13

CONSTANT FORCE. CRYSTALS PARALLEL

Temperature	Weight	Duration	Deformation	Recovery	Percentage
−9°.2C.......	5 gm.	0^h55^m	$0^\circ\ 44'$	$0^\circ\ 4'$	100
−8.6........	44	1 45	0 10	0 10	100
−6.3........	88	15 10	1 38	1 15	77
−5.8........	132	15 0	2 48	1 6	39
−7.3........	229	29 30	7 12	1 21	19
−5.7........	220	12 40	5 24	0 48	15
−7.1........	308	48 0	11 11	1 16	12
−4.4........	440	13 0	13 38	1 56	14
−5.7........	660	18 0	20 36	2 8	10

INCREASING FORCE. CRYSTALS TRANSVERSE

Temperature	Increased Rate	Duration	Final Weight	Deformation	Recovery	Percentage
− 4°.5C.....	.44 gm./min.	13^h20^m	352 gm.	$7^\circ\ 12'$	$1^\circ\ 24'$	19
− 4.7......	.88	5 0	264	3 20	1 10	35
− 4.6......	1.76	2 5	222	2 2	0 54	44
−10.4......	1.76	6 15	660	8 58	2 46	31
− 5.0......	2.2	3 20	440	6 8	1 46	29
− 5.4......	4.4	2 10	572	7 12	1 58	27
− 6.3......	8.8	1 5	572	5 28	1 42	31
− 4.3......	8.8	1 5	572	5 56	1 30	25
− 6.0......	22.0	0 30	660	5 30	0 58	18
− 5.7......	44.0	0 15	660	4 0	1 12	30

INCREASING FORCE. CRYSTALS PARALLEL

Temperature	Increased Rate	Duration	Final Weight	Deformation	Recovery	Percentage
− 7°.8C....	.88 gm./min.	3^h35^m	189 gm.	$0^\circ\ 57'$	$0^\circ\ 25'$	44
− 5.5......	1.76	4 10	440	6 30	2 0	31
− 9.0......	44.0	0 15	660	1 27	0 36	41

The amount of recovery in percentage of the deformation is shown in Figures 9 and 10. For small forces the recovery is complete. As the amount of force which has been used increases the percentage of recovery decreases rapidly until the formation amounts to about four degrees. After this its decrease becomes slower and it approaches gradually to the zero line. Thus it is seen that the transition from the stage of nearly perfect elasticity to that of partial elasticity takes place very slowly in the cases both when the torsion is parallel to the optic axis of the constituent crystals and when it is parallel to the basal planes.

BENDING EXPERIMENTS

The observations on torsion of ice just presented have suggested that an aggregate of parallel crystals of ice is more easily deformed by a shearing force parallel to the axes of the constituent crystals than parallel to their basal sections. We have also seen that Young's modulus, or proportionate resistance to elongation which is generally determined by the method of bending, is greatest when the deforming force bending the test bar acts parallel to the basal sections of the constituent parallel crystals. The next step was to test the matter further by observing the behavior of bars of differently orientated crystals when bent beyond the limit of elasticity. It was desirable to try the experiment with bars of different sizes compared to the constituent crystals. But the temperature of laboratory was not favorable for the use of larger bars with larger deforming force. Bars of moderate size with rectangular cross-sections were therefore prepared with different orientations of crystals. Each bar was supported at both ends by knife-edges and a weight of 100 gm. was hung from the middle point. The bending was measured by the lowering of this central portion. In Figure 11 some of the results of these observations are given, the data for the bars being given in the following table:

Bar	Optic Axes	Span	Breadth	Thickness	Temperature
a........	Horizontal	9.0 cm.	.90 cm.	.60 cm.	−5°8 C. to −2°0 C.
c........	Parallel	9.0	.90	60	−5.8 to −3.0
b........	Vertical	9.0	.60	.58	−2.6 to −0.2
c........	Parallel	9.0	.60	.58	−2.6 to 0.0

The first specimen bent much more easily than the second and broke at the point *A* in the figure. The same applies to the third and fourth specimens.

In another case, three test pieces, each 0.80 cm. wide and 0.50 cm. thick, were supported with an 8.0 cm. span between the supports. The lowering of the central part by 100 gm. in the first eight hours was 2.68 cm., 0.2 cm., and 1.22 cm. respectively for the different orientations *a*, *b*, and *c*. A part of the weight hanging from the second bar was found supported from beneath by accident so that its bending is not comparable with others. The temperature during this observation ranged from −5°0 to −2°7 C.

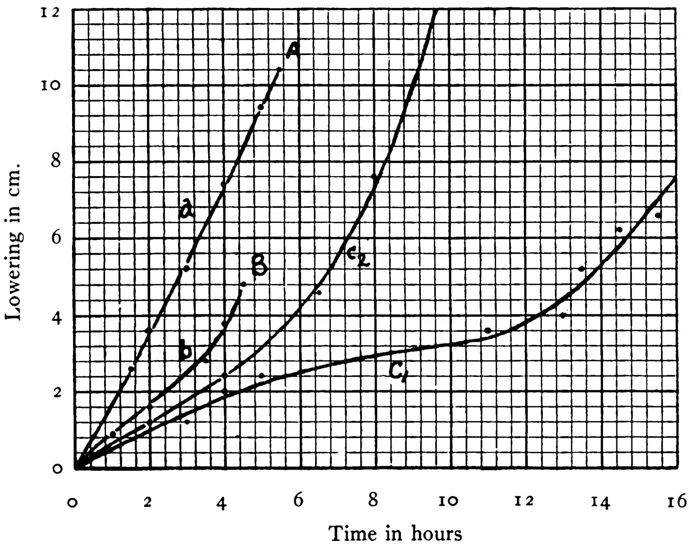

FIG. 11.—Relative ease of bending of ice bars with different orientations of crystals.

It is clear from these observations that ice bars with their constituent crystals perpendicular to the length, bend and break more easily than the bar with crystals parallel to the length. In the case of observation for the bars *b* and *c*, the temperature became so high that the bar suffered from pressure melting at the knife-edges, thus preventing free movement. On account of this, the result may not be much relied upon and consequently the relative ease of bending of the bars *a* and *b* is not determined. The other observation stated above also failed to give any idea about this.

As to the nature of deformation of ice comprising of an aggregate of crystals, many authors have claimed that it depends upon the behavior of a single crystal. It is stated that in a case of bending of an ice bar consisting of several crystals, most of the bending had taken place in one of the crystals lying with its crystal axis nearly horizontal and approximately parallel to the length of the bar.[1] This is understood to mean that in such a case movement along the gliding planes of an ice crystal parallel to its basal plane is more effective than movements along the contact surfaces of adjacent crystals. If this theory is applicable to the present case the third bar in which the gliding planes were transverse to the long dimension should bend most easily. But quite to the contrary, bar of orientation *c*, instead of bending most readily, suffered the least bending of any of the three types of orientation.

As to the mechanism of bending within the limit of perfect elasticity it is generally understood by physicists that bending of a bar is caused by shortening of the concave side and elongation of the convex. When bending becomes larger than this, it is difficult to solve the case as a simple mechanical problem. It is probable in such a case that the portion near the point of application of force is subjected to bending, while the other parts are elongated with some degree of sliding at the knife-edges. The mechanism at the bending-point will depend upon the structure of the material. If it is deformable more easily in one direction than in another the problem becomes complicated. The resulting deformation in such a case will be the combination of that effect with the result of shortening and elongation phenomena. The experimental fact above described that bars of type *c* bend less easily than the others suggests that contact surfaces between adjacent crystals play greater rôle than the so-called gliding planes parallel to the basal plane.

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[1] R. S. Tarr and J. L. Rich, *Zeits. f. Glets.*, Vol. VI (1911), p. 236.

MICROSCOPIC EVIDENCES

The theory that an ice crystal is composed of thin laminae parallel to its basal plane gives rise to the conclusion that when the crystal is bent by force perpendicular to its optic axis, it will show simultaneous extinction at the bent and unbent portions under the petrographic microscope.[1] Tarr and Rich[2] have described the case in which this optical property of the bent bar was not changed, as well as when it was changed. In the latter case the original optic axis was either parallel or perpendicular to the length of the bar, and if we assume the latter as the case, it is in contradiction to the idea that movement along the gliding planes controlled the bending.

The present writer examined the bent parts of an ice bar and the results were very suggestive, showing facts which seem not to agree with former ideas. A bar of ice with crystals parallel to its length was bent under certain stress. When the bent portion was thinned down and examined under the microscope with the Nicols crossed, it clearly showed an extinction strip across the bar, which moved along the bent portion as the stage was turned. Wanting to be sure about this, the writer asked Professor R. T. Chamberlin to see it and he recognized the same fact with certainty. The test piece in this case consisted of one main crystal with small portions of other crystals on either side. The same fact was observed in one more case but with the oncoming of warm weather these two were the only trials which it was possible to undertake in the present investigation.

Examinations without crossed Nicols revealed another important fact. The test specimen consisted of parallel crystals whose optical axes were horizontally transverse to the length. The bent portion was examined under microscope so as to see the side of the bar, i.e., the basal planes of the crystals. In the field of the microscope, it was found that faint but distinct straight lines nearly parallel to each other had developed on the sections of the crystals. The boundaries of the crystals were zigzag and sometimes the straight lines developed in the crystals were observed to start from the angular points of these zigzag boundaries. The direction of the parallel straight lines was not the same in different crystals. Sometimes apparently two systems of these straight lines developed in one crystal, nearly perpendicular to each other. When the unbent portion of the same bar was examined in the same way, the crystals were found to be bounded by very smooth boundaries and no straight lines in their sections were visible. In another case, the same crystals were identified before and after bending took place. The same contrast of the disturbed and undisturbed crystals was observable in this case.

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[1] R. S. Tarr and O. D. von Engeln, *Zeits. f. Glets.*, Vol. IX (1915), p. 111.

[2] R. S. Tarr and J. L. Rich, *Zeits. f. Glets.*, Vol. VI (1911–12), p. 235.

At the time of these observations, the equipment for petrographic photography was not available for the writer. He was obliged to content himself with very careful sketches of these phenomena as they appeared to him. These are shown in Figures 12 and 13. Since these figures represent the sections nearly parallel to the base, the straight lines in the sections must be considered to show the development of a system or two of parallel planes parallel to the optic axis. Uniform extinction was generally observed throughout each individual crystal, but in some crystals portions divided by the straight lines showed slight difference in extinction.

Bearing upon the question of straight lines in the section, Tarr and Rich[1] describe one case in which phenomenon of the sort were observed. They regarded it as notable, however, that this was the only one of their experiments in which bending took place by shearing. Their experiment was about the bending of a single crystal of glacier ice whose optic axis was parallel to the supporting edge. In the present investigation, the same phenomena was observed whenever the constituent crystals were perpendicular to the bending plane. It has already been stated that deformation either by torsional, or by bending, stress took place least easily when the force was applied parallel to the basal section of the constituent crystals. This was thought to suggest that in the deformation of an aggregate of parallel crystals, the contact surfaces between adjacent crystals probably have played an important

[1] R. S. Tarr and J. L. Rich, *Zeits. f. Glets.*, Vol. VI (1911–12), p. 243.

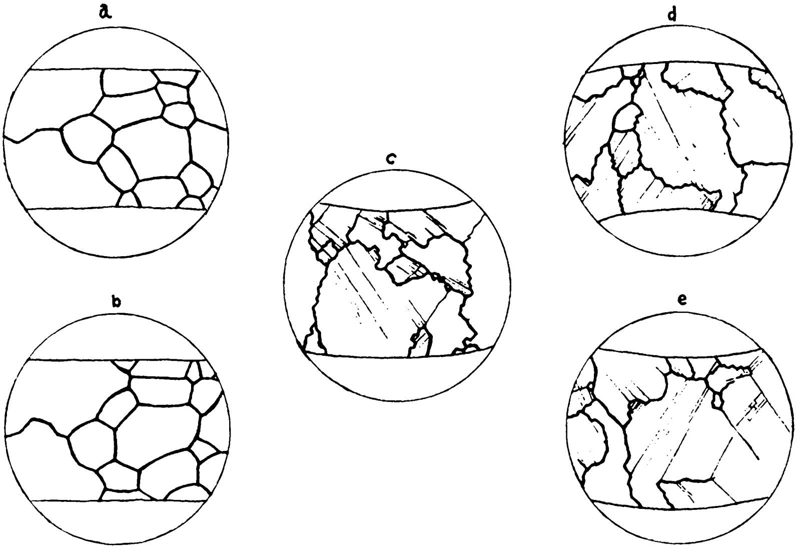

FIG. 12.—Microscopic appearances of ice crystals after bending. *a* and *b*: unbent portions. *c*, *d*, and *e*: bent portions.

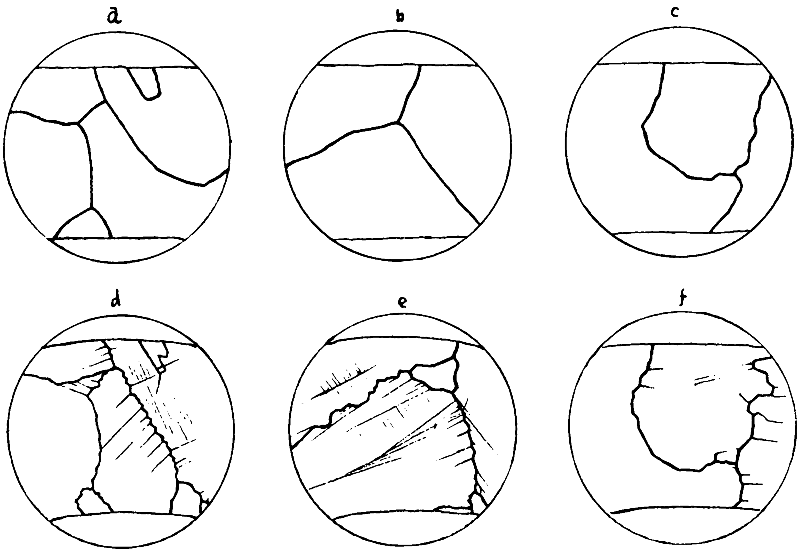

FIG. 13.—Microscopic appearances of ice crystals before and after bending. *a*, *b*, and *c*: before bending. *d*, *e*, and *f*: after bending.

part if the idea of McConnell and Mügge is correct. The development of the planes parallel to the optic axis suggests that an ice crystal has a greater tendency toward deformation parallel to the optic axis than perpendicular to it. To what degree this property of an ice crystal and the contact surfaces are concerned in the deformation of ice is to be decided by further study.

CONCLUSIONS

In some specimens of ice a sharply defined elastic limit was noted, though in other cases it was not so clearly shown.

The modulus of rigidity of ice, when the crystals are perpendicular to the axis of the test piece, is very small compared to that of metals, and is about 2×10^6 c.g.s. There is a slight indication that it is greater when the shearing is parallel to the base of the constituent crystals than when it is perpendicular.

The Young's modulus is also very small compared to that of metals. It is largest when the crystals are parallel to the length of the test piece, and has the numerical value about 20×10^6 c.g.s.

Elastic fatigue was marked after repeated torsion. On account of the fact that it was often necessary to use certain bars in successive experiments during which they suffered from different amounts of fatigue, it was difficult to compare the results bearing on ice bars with crystals parallel and perpendicular to the length of the test piece. Still there were some indications that beyond the limit of elasticity the former orientation was stronger than the latter against torsion. In the case of bending experiments, this was clearly shown.

The torsional deformations both by constant and by increasing forces were observed and the result is shown by curves, though no mathematical conclusions were made. The recovery curves showed that the observation was approaching the stage where no recovery would take place after removal of the force.

When an ice bar with crystals parallel to its length was bent, the bent portion showed the change of optical character, the extinction swinging around the curve. In each crystal when the bent specimen consisted of parallel crystals horizontally across it, parallel straight lines were observed.

These facts seem to show that gliding planes parallel to the base of each crystal are not the controlling factor in the deformation of ice and probably are not even an important factor. But instead, adjustments along the contact surfaces of adjacent crystals and perhaps the development of planes of weakness in the constituent crystals parallel to their long axis seem more effective in the process of deformation.